Unidirectional edge states in topological honeycomb-lattice membrane photonic crystals

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Abstract: Photonic analogs of electronic systems with topologically non-trivial behavior such as unidirectional scatter-free propagation has tremendous potential for transforming photonic systems. Like in electronics topological behavior can be observed in photonics for systems either preserving time-reversal (TR) symmetry or explicitly breaking it. TR symmetry breaking requires magneto-optic photonics crystals (PC) or generation of synthetic gauge fields. For on-chip photonics that operate at optical frequencies both are quite challenging because of poor magneto-optic response of materials or substantial nanofabrication challenges in generating synthetic gauge fields. A recent work by Ma, et al. [Phys. Rev. Lett. 114, 223901 (2015)] based on preserving pseudo TR symmetry offers a promising design scheme for observing unidirectional edge states in a modified honeycomb photonic crystal (PC) lattice of circular rods that offers encouraging alternatives. Here we propose through bandstructure calculations the inverse system of modified honeycomb PC of circular holes in a dielectric membrane which is more attractive from fabrication standpoint for on-chip applications. We observe trivial and non-trivial bandgaps as well as unidirectional edge states of opposite helicity propagating in opposite directions at the interface of a trivial and non-trivial PC structures. Around 1550nm operating wavelength ~55nm of bandwidth is possible for practicable values of design parameters (lattice constant, hole radii, membrane thickness, scaling factor etc.) and robust to reasonable variations in those parameters.

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References and links


1. Introduction

Topologically non-trivial behavior in electronic systems has been studied intensively following the discovery of quantum hall effect [1, 2] (QHE) in two dimensional electron gas (2DEG) at low temperatures in an external magnetic field that breaks time-reversal symmetry. A characteristic feature of the topological protection is the one-way conducting electronic edge-states that propagate scatter-free even in the presence of large impurities. This has opened up possibilities in quantum computation for achieving long lived qubits in decohering environments. Interestingly, it was theoretically shown that such behavior can be ported to
photonic crystals [3, 4] composed of magneto-optic materials. Since then there have been several theoretical proposals and experiments demonstrating topological photonics [5] in photonic crystals [6–15] and metamaterials [16–22]. These include Floquet systems [9], time-reversal symmetry breaking systems using either external magnetic fields [12, 23] or synthetic gauge fields [14]. Topologically non-trivial properties such as unidirectional scatter free propagation can be game changing for on-chip photonics. Furthermore, for photonics based quantum communications [24] loss free transport of single photon states can be quite important particularly because they cannot be amplified [25]. For on-chip photonics applications explicitly breaking time-reversal symmetry either using magneto-optic materials or using synthetic gauge fields, though robust, can be very challenging. Most materials have very low magneto-optic coefficients at optical frequencies and therefore require high magnetic fields. On the other hand, generation of synthetic gauge fields require some form of phase coherent dynamic modulation of PC properties which pose fabrication challenges at optical frequencies. Notwithstanding some recent examples of chipscale optical frequency demonstration [26–28] alternate approaches to achieving topological properties can be quite attractive as it offers design flexibility. A recent work [6] has proposed a scheme wherein a simple modification to a well-known honeycomb lattice photonic crystal that preserves pseudo time-reversal symmetry can exhibit helicity dependent unidirectional propagation of edge states. The strategy basically involves opening up of a band gap about a Dirac point by compressing or expanding the honeycomb lattice. This was shown theoretically for a photonic crystal system composed of an array of infinitely long dielectric cylinders. However, for on-chip photonics applications photonic crystal composed of hole arrays in a dielectric membrane is more practical and suitable for fabrication. Such a dielectric membrane photonic crystal composed of a honeycomb lattice of triangular holes have also been studied recently [7]. However, triangular holes can pose fabrication challenges at feature dimensions (10s to ~100nm) required for optical frequencies with regards to shape accuracy. A circular hole array is less challenging from fabrication perspective and therefore more desirable for practical reasons. Additionally, rounding off of the triangular corners may also lead to circular features. However, it has been suggested [7] that it is not possible to open a gap at the doubly degenerate Dirac point with circular holes for any ratio of expansion/compression the six-hole unit cell. In the following we show that while that may be true for some specific values of the circular hole radii it is indeed possible to find a range of hole radii for which one can open a gap at the Dirac point for both the expanded and compressed lattices. Likewise, it is possible to observe helicity dependent edge states at the interface of a topologically non-trivial and topologically trivial lattices. The circular hole array based membrane photonic crystal provides a simpler and more practical way to fabricate and experimentally demonstrate this behavior at optical frequencies on-chip.

2. Honeycomb lattice photonic crystal

The design we will discuss in the following is based on a hole array photonic crystal in a high refractive index membrane. This is an attractive architecture for applications involving optical communication wavelengths (λ~1550nm) utilizing semiconductor materials such as III-V and silicon-on-insulator that are quite amenable to nanofabrication and integration. The high refractive index (n = 3-3.5) of these materials provide excellent in-plane light confinement and have been the basis of many photonic device designs ranging from waveguides, cavities and light sources [29–38]. We will consider a honeycomb lattice photonic crystal as shown in Fig. 1(a). The unmodified honeycomb photonic crystal can be obtained by removing every third “photonic atom” from the lattice site of a triangular lattice with a lattice period of ‘R’. This can also be regarded as a triangular lattice with a period ‘a = 3xR’ of a six atom basis unit cell where each photonic atom is located at the corner of the hexagon of side or radius R in each unit cell. Ref [6] describes a design where the photonic atoms consist of infinitely tall cylinder composed of a high refractive index material. For the unmodified honeycomb lattice
this results in a photonic bandstructure with Dirac cones at the K.K’ symmetry points in the first Brillouin zone in a two-atom basis for the transverse magnetic (TM) polarization (electric field $E$ parallel to the cylinder axis). In the six-atom basis the K and K’ points fold into a doubly-degenerate Dirac points at the Gamma point. Since a photonic atom in a PC can either be a high dielectric material in a low refractive index dielectric background media or low dielectric inclusion (or a hole) in a high dielectric background media one may expect similar topological properties for the latter case. Due to the duality of the system one would expect the topological properties observed for high dielectric rod system would simply be swapped for TE polarization instead of TM wherein the role of electric field ($E$) will be swapped with the magnetic field ($H$). Furthermore, instead of infinitely long cylindrical holes in an infinitely thick dielectric system we will consider here a PC with cylindrical holes in a high dielectric slab or a membrane of a finite thickness. While this would be a more practical design from the perspective of fabrication suitable for optical frequency on-chip photonic applications, it is important to systematically explore if such a system does exhibit topological behavior. In this case, we will consider photonic band structure with ‘TE-like’ polarization where in electric field is completely in-plane at the mid-plane of the dielectric membrane with the magnetic field perpendicular.

The various salient parameters of the honeycomb lattice are its periodicity of ‘$a$’, hole radius ‘$r$’, ratio of the lattice constant to the distance from lattice center to the center of the photonic atom ($a/R$) and the height of the dielectric membrane ‘$h$’ [Fig. 1(a)]. First, we evaluate the band structure for the unmodified honeycomb lattice ($a/R = 3.0$) for a hole radius $r = 0.13a$ and Si membrane height $h = 0.25a$. We calculated the band structures using the Lumerical® FDTD solutions finite difference time domain software. The band structure reveals a doubly degenerate Dirac point at a reduced frequency $a/\lambda = 0.5706$ [Fig. 1(b)]. The membrane is assumed to be composed of Si with a dielectric constant ($\varepsilon_d \sim 11.5$). The results discussed will hold quite well for other semiconductor material systems including GaAs, InP that possess similarly high refractive indexes at optical frequencies.
3. Discussion

A photonic band gap can be opened up by expanding or compressing the hexagon (increasing or decreasing the radius $R$) in each unit cell in the six-atom basis. Upon shrinking the unit cell making the scaling factor $a/R = 3.1$ [Fig. 2(a)] results in a topologically trivial PC with bandgap opening up between the reduced frequencies of 0.5626 and 0.5749 with a gap of 0.0123 [Fig. 2(b)]. The magnetic field at the symmetry plane ($z = 0$) is primarily along the $z$ direction (normal to the membrane plane) as one would expect for the hole array PC. At the top of the lower band at the $\Gamma$ point the Re($H_z$) field distribution has $p_x$ like mode symmetry whereas at the bottom of the upperband at the $\Gamma$ point the mode symmetry is $d_{xy}$ [Fig. 2(c)].

The $p_x$ mode can be identified by the two lobes in each side about the $y$ axis (analogous to the two-lobed atomic orbitals) seen within the yellow dashed rectangles enclosing a honeycomb lattice site. Likewise, the $d_{xy}$ mode is identified by the four lobes observed within the yellow dashed rectangle enclosing the honeycomb lattice site, one in each quadrant similar to the atomic $d_{xy}$ orbitals. However, the expanded version results in a topologically non-trivial photonic crystal which displays Kramer’s degeneracy due to the inherent TR symmetry of Maxwell’s equation as well as the C6 point group crystal symmetry exhibiting up and down pseudo spins. Upon expanding the unit cell to $a/R = 2.9$ [Fig. 2(d)] we once again create a
gap but between the reduced frequencies of 0.5587 and 0.5751 resulting in gap of 0.0164 which is slightly larger than the trivial case [Fig. 2(e)]. In this case however the $Re(H_z)$ field distribution at the $\Gamma$ point exhibits an inversion i.e., the lower band has $d_x$ symmetry while the upper band has $p_z$ symmetry indicating a change in the band topology resulting in a topologically non-trivial PC [Fig. 2(f)].

![Graphs showing midgap frequency, gap size, and gap/midgap ratio variations for different hole radii and membrane thicknesses.](image)

Fig. 3. (a) Plot of midgap frequency for varying hole radii ‘r’ for the 4 different cases of $a/R$ ratios. (b) Gap size in reduced frequencies ($a/\lambda$) for varying hole radii for the 4 different $a/R$ ratios. (c) Gap/midgap ratios for varying hole radii. (d) Midgap frequency for varying membrane thicknesses ($h$) for the 4 different $a/R$ ratios. (e) Gap size for varying membrane thicknesses. (f) Gap/midgap ratios for varying membrane thicknesses.

While this is encouraging, in the context of on-chip optical frequency photonics it is important and useful to explore the range of PC parameters for which one can expect topologically trivial and non-trivial bandgaps and thus, evaluate the robustness of this design from the standpoint of nanofabrication. To that end we have systematically studied the effect on the bandgaps by varying the hole radius for both topologically non-trivial ($a/R = 2.8, 2.9$) and topologically trivial PCs ($a/R = 3.1, 3.2$) [Figs. 3(a)-(c)]. The hole radii were varied from $r = 0.11a$ to $0.15a$ in steps of $0.01a$. In all cases the midgap frequency monotonically increases [Fig. 3(a)] with radius which is to be expected as the effective refractive index of the PC is decreasing. The midgap frequency is the midpoint between maximal frequency point of the lower band and the minimal frequency of the upper band forming the photonic bandgap.

The band gap vs. hole radius plot on the other hand exhibits a shallow maximum [Fig. 3(b)]. For the trivial cases ($a/R = 3.1$ and 3.2) the gap starts small (~0.005) at $r = 0.11$ peaking to ~0.012 around $r = 0.13-0.14a$. For a radius of 0.15a the gap vanishes. (The bandgap appears negative since the lowest point in the upperband is below in frequency than the highest point in the lowerband). For the non-trivial cases ($a/R = 2.8$ and 2.9) the value of the gaps are larger than the trivial cases i.e., 0.023 and 0.0182 at $r = 0.14$ for $a/R$ of 2.8 and 2.9 respectively. Furthermore, the gap-to-midgap ratio which provides a normalized the gap size with respect to midgap frequency also follows a trend similar to the gap [Fig. 3(c)] but
exhibits a somewhat flatter peak between $r = 0.13a$ and $r = 0.14a$. This while expected also indicates a potential choice of a robust operating point.

The trend observed here is typical for bandgaps in PCs since for small radii and large radii the scatter strength is small resulting in weaker coherent scattering effects and for very large radii ($r \sim 0.5a$) the PC periodicity itself disappears. The point to note here however, is that the loss of gap does occur at much smaller radii value ($r \sim 0.15a$) compared to typical PC structures due to the unique geometry of the modified honeycomb PC structure. Two different types of hole overlaps for the trivial and non-trivial cases can occur that can potentially destroy the PC structure and the bandgap. In the trivial case, the holes could overlap within the unit cells [Fig. 2(a): $\delta = R-2r \rightarrow 0$] while for the non-trivial case the overlaps could occur across the neighboring unit cells [Fig. 2(e): $\Delta = a-2R-2r \rightarrow 0$. This will essentially restrict the range of $a/R$ ratios that will be accessible. Furthermore, fabrication constraints likely can impose additional restrictions on the hole radii even prior to any onset of hole overlap.

Another parameter that can potentially affect the bandstructure is the membrane thickness. Figures 3(d)-3(f) shows the midgap frequency, bandgap and gap-to-midgap ratio as a function of membrane thickness which is another parameter that can vary, for different $a/R$ ratios. As expected the bandgap center frequency monotonically decreases with the thickness of the dielectric membrane as the effective refractive index increases. Bandgap drops slightly as the membrane thickness is increased from 0.2a to 0.4a while the gap-to-midgap ratio slightly increases and tends to flatten out as $h$ is increased. The trend towards saturation of the gap-to-midgap ratio (more apparent) and of the midgap frequency (less apparent) points to the limiting case of infinitely long cylindrical holes in high dielectric matrix or the pure two dimensional case.

For a center wavelength of operation around $\lambda \sim 1550$nm, assuming a nominal hole radius of $r = 0.13a$ and height $h = 0.25a$, the photonic bandgap for the trivial structure can be around 30nm while for the non-trivial structure can be as much as 60nm. This shows that we can achieve a practically useful bandwidth of operation which is important from the perspective of applications. Likewise, for the scaling ratios ($a/R$) investigated here the lattice constant ‘$a$’ ranges from $\sim 860$nm to $\sim 880$nm with corresponding hole radii in the range of $\sim 110$ nm. This results in a smallest feature size $\sim 25-60$nm (neck region between two neighboring holes, $\Delta = a-2(R-r)$) which are well within current fabrication capabilities. The above analysis thus indicates that the approach offers a practicable design latitude necessary for fabrication.

### 3.1 Topologically trivial and non-trivial interface

In this section we will examine the helicity dependent one-way propagation at the interface between the topological and trivial PCs. We will consider two different interfaces (namely, the armchair and zig-zag interface) between the topological and the trivial lattices with the topologically non-trivial and the trivial structure on the left and the right respectively. The lattice constants as well as the hole radii ($r = 0.13a$) are taken to be the same for both sides with the membrane thickness $h = 0.25a$. The topologically non-trivial side has $a/R = 2.9$ and the trivial side has $a/R = 3.1$. 
Fig. 4. (a) Schematic of the zig-zag interface between topological and trivial lattice for a/R 2.9, 3.1 respectively. (b) Plot of the projected band structure along the interface ‘y’ direction showing two unidirectional gap states. (c) Average power flow at the interface corresponding $k_y = -0.05(2\pi/a)$ (indicated by red up arrow in (b)) in the upward or positive ‘y’ direction. (d) Average power flow at the interface corresponding $k_y = 0.05(2\pi/a)$ (indicated by blue down arrow in (b)) in the downward or negative ‘y’ direction.

For these values the midgap frequencies are aligned relatively close to each other around 0.568 in reduced frequencies enabling an optimal overlap of the bandgap. The zigzag interface is obtained when the corners of the hexagons are in line [Fig. 4(a)] with periodicity in the horizontal direction (perpendicular to the interface) equal to the lattice constant $a$. Figure 4(b) shows the projected band structure ($\omega$ vs $k_y$) along the vertical direction where there are extended states above and below the reduced frequencies of 0.570 and 0.548 respectively. There are two distinct modes within the gap between the two frequencies connecting the upper and the lower bands exhibiting unidirectional propagation. For the wavevector ($k_x$) corresponding to $-0.05(2\pi/a)$ (indicated by the red ‘up arrow’ we observe that the Poynting vector distribution [Fig. 4(c)] clearly indicates a steady state power flow in the upward or positive y direction.

On the other hand, for $k_y = 0.05(2\pi/a)$ we observe a steady state power flow in the downward or negative y direction [Fig. 4(d)]. In both cases, at $a/\lambda \sim 0.56$ corresponding to these wavevectors, the power flow occurs though the holes at the interface. For the armchair interface [Fig. 5(a)] the periodicity is $a$ along the interface direction and we once again observe a bandgap between 0.570 and 0.548 with unidirectional gap states [Fig. 5(b)] connecting the lower and upper bands. Figure 5(c) shows power flow in the positive y direction for $k_y = -0.06(2\pi/a)$ and Fig. 5(d) shows power flow in the negative y direction for $k_y = 0.06(2\pi/a)$ corresponding to $a/\lambda \sim 0.56$. Poynting vector field distribution at the interface is different from the zig-zag interface case but is once again concentrated in the holes. Therefore, we observe that unidirectional modes exist for both types of simple interfaces between the topologically trivial and non-trivial interfaces.

Assuming a lattice constant ‘a’ of 870nm this enables unidirectional modes between the wavelengths of ~1525nm to 1590nm equivalent to a bandwidth of ~55nm. The pertinent radius, membrane thickness and minimum feature sizes are ~110nm, ~220nm and ~25nm respectively. This indicates from the perspective of potential on chip silicon photonics applications this design can be fabricated and will exhibit a practicable operational bandwidth. While this example demonstrates specific a/R ratios, hole radius and membrane thickness the analysis of the previous section indicates that there is also reasonable latitude available in these parameters to design structures suitable to requirements.
4. Summary

In summary, we have presented a honeycomb-lattice membrane photonic crystal design with pseudo time-reversal symmetry demonstrating unidirectional helical edge states. Specifically, our proposed honeycomb PC design is composed of circular holes in a high dielectric membrane that can be readily implemented in a straightforward manner for fabrication in silicon-on-insulator platform. We demonstrated with a systematic analysis of the bandstructures for the compressed (trivial) and the expanded honeycomb PC (topological) there exists a practicable range of parameters \((r, h, a/R)\) where structures can be created with unidirectional edge states. In particular, use of circular holes in a dielectric membrane instead of other shapes or infinite rod structure proposed previously can make fabrication considerably less challenging. We show that unidirectional edge states can be observed for both armchair and zig-zag interfaces between topological and trivial lattices which for operation around telecom wavelengths of 1550nm provides a practicable operational bandwidth of ~55nm. We believe this design scheme offers a more practical pathway for implementing topological photonics at optical frequencies on a chip-scale.

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